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SOLIDIFICATION OF A KILLED-STEEL INGOT

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An approximate solution is presented for the problem of the thickness of the diffusion boundary layer on solidification of a killed-steel ingot under conditions of natural thermal-melt convection.

The distribution of a soluble admixture in the case of normally directed crystallization, as is well known, is given by the effective distribution coefficient determined by Burton-Prim-Slichter [1]:

$$k = \frac{k_0}{k_0 + (1 - k_0) \exp(-\Delta)}, \quad (1)$$

where $\Delta = f\delta/D$, while the equilibrium distribution coefficient $k_0 = C_s/C_0$.

Of the three fundamental parameters f , δ , and D that determine the normalized rate of growth Δ and, consequently, the effective distribution coefficient k , the growth rate f , and the diffusion factor D in the liquid are generally known. If the conditions of the liquid flow for the solidification of the alloy are known, it is theoretically sometimes possible to calculate the thickness δ of the diffusion boundary layer. Solutions have been obtained for the thickness δ of the diffusion layer at the end of a single-crystal axis on vertical extraction from the melt [1] and with concentrated natural convection of the melt [2], defined by the differing density of the liquid in the boundary layer enriched by an alloying admixture, and in the main mass of the melt. Analogous solutions for heterogeneous transformations in a liquid, not accompanied by heat liberation, are given by Levich [3].

As we demonstrated earlier [4-6], the solidification of a killed-steel ingot is accompanied by the natural thermal convection of the melt, when the colder layers of the metal at the crystallization front, enriched with solid-phase nuclei, sink down, displacing the warmer but less dense layers of the melt upward along the axis of the ingot. The calculation of the diffusion-layer thickness under the conditions of natural thermal convection merits attention.

The integral balance equation for the admixture in the volume of the boundary layer [2]

$$v_n C_1^0 = \frac{d}{dx} \int_0^l C_1 v dy + f C_s, \quad (2)$$

where the upper integration limit l is greater than both the thickness of the diffusion (δ) and the hydrodynamic (δ_0) boundary layers.

If we neglect the local values in liquid density and the difference between the densities of the liquid and solid phases, the velocity v_n of the liquid influx to the boundary layer normal to the solid-liquid interface is defined by the equation for the conservation of mass

$$v_n = \frac{d}{dx} \int_0^l v dy + f. \quad (3)$$

From (2) and (3) we obtain

$$f(C_1^0 - C_s) = \frac{d}{dx} \int_0^l (C_1 - C_1^0) v dy. \quad (4)$$

The upper integration limit l can be replaced by δ , since when $y > \delta$, $C_1 - C_1^0 = 0$.

Thus,

$$f(C_1^0 - C_s) = \frac{d}{dx} \int_0^\delta (C_1 - C_1^0) v dy. \quad (5)$$

For an approximate solution of (5) let us express the velocity of the liquid flow in the case of natural thermal convection [7] and the distribution of the admixture concentration in the boundary layer [1] in the following form:

$$v = v_1 \frac{y}{\delta_0} \left(1 - \frac{y}{\delta_0}\right)^2 \text{ for } 0 \leq y \leq \delta_0, \quad (6)$$

where [7]

$$v_1 = 5.15v \left(0.952 + \frac{v}{a}\right)^{-\frac{1}{2}} \left(\frac{g\beta\Theta}{v^2}\right)^{\frac{1}{2}} x^{\frac{1}{2}}, \quad (7)$$

$$\delta_0 = 3.93 \left(\frac{v}{a}\right)^{-\frac{1}{2}} \left(0.952 + \frac{v}{a}\right)^{\frac{1}{4}} \times \left(\frac{g\beta\Theta}{v^2}\right)^{-\frac{1}{4}} x^{\frac{1}{4}} = \lambda x^{\frac{1}{4}}, \quad (8)$$

$$C_i = C_i^0 \frac{k_0 + (1-k_0) \exp\left(-\frac{fy}{D}\right)}{k_0 + (1-k_0) \exp\left(-\frac{f\delta}{D}\right)} \quad \text{for } 0 \leq y \leq \delta. \quad (9)$$

From (7) and (8)

$$v_1 = 79.5 \frac{ax}{\delta_0^2}. \quad (10)$$

Having substituted (6), (9), and (10) into (5) and having taken into consideration the expression $k = C_S/C_1^0$ from (1), we obtain

$$\frac{f}{k_0 \exp \Delta + 1 - k_0} = 79.5 a \times \times \frac{d}{dx} \left\{ \frac{x}{\delta_0^2 (k_0 \exp \Delta + 1 - k_0)} \int_0^\delta y \left(1 - \frac{y}{\delta_0}\right)^2 \times \left[\exp\left(\Delta - \frac{fy}{D}\right) - 1 \right] dy \right\}. \quad (11)$$

Let us calculate the integral in the right-hand part of (11)

$$\int_0^\delta y \left(1 - \frac{y}{\delta_0}\right)^2 \left[\exp\left(\Delta - \frac{fy}{D}\right) - 1 \right] dy = \left(\frac{D}{f}\right)^2 \left[\left(\exp \Delta - \frac{\Delta^2}{2!} - \Delta - 1\right) - \frac{4\sigma}{\Delta} \left(\exp \Delta - \frac{\Delta^3}{3!} - \frac{\Delta^2}{2!} - \Delta - 1\right) + \frac{6\sigma^2}{\Delta^2} \left(\exp \Delta - \frac{\Delta^4}{4!} - \frac{\Delta^3}{3!} - \frac{\Delta^2}{2!} - \Delta - 1\right) \right], \quad (12)$$

where $\sigma = \delta/\delta_0 < 1$, or

$$\int_0^\delta y \left(1 - \frac{y}{\delta_0}\right)^2 \left[\exp\left(\Delta - \frac{fy}{D}\right) - 1 \right] dy \approx \left(\frac{D}{f}\right)^2 \left[\exp \Delta - \frac{\Delta^2}{2!} - \Delta - 1 \right]. \quad (13)$$

Having substituted (13) into (11), we obtain

$$\frac{f^3 \lambda^3}{79.5 a D^2} = (k_0 \exp \Delta + 1 - k_0) \times \quad (14)$$

$$\times \frac{d}{dx} \left[\frac{x^{\frac{1}{4}} \left(\exp \Delta - \frac{\Delta^2}{2!} - \Delta - 1 \right)}{k_0 \exp \Delta + 1 - k_0} \right].$$

The solution of the differential equation (14) is sought in the form

$$\Delta = \sqrt[3]{6A} x^{\frac{1}{4}} U(x), \quad (15)$$

where

$$A = \frac{f^3 \lambda^3}{79.5 a D^2}; \quad (16)$$

$$U(x) = a_0 + a_1 x^{\frac{1}{4}} + a_2 x^{\frac{1}{2}} + \dots \quad (17)$$

Substituting (15) into (14) and expanding $\exp \Delta$ in series, from the recurrence relationship for the series coefficients a_0, a_1, \dots by the method of successive approximations we find

$$a_0 = 1, \quad a_1 = -\frac{1}{60} (5 - 4k_0) \sqrt[3]{6A} \text{ etc.}$$

The sought solution for (14) is expressed in the form of the series

$$\Delta = \sqrt[3]{6A} x^{\frac{1}{4}} \left(1 - \frac{5 - 4k_0}{60} \sqrt[3]{6A} x^{\frac{1}{4}} + \dots \right), \quad (18)$$

or

$$\Delta = 0.42 f (aD^2)^{-\frac{1}{3}} \delta_0 \times \left[1 - 0.42 \frac{5 - 4k_0}{60} f (aD^2)^{-\frac{1}{3}} \delta_0 + \dots \right]. \quad (19)$$

With a sufficiently low growth rate f we can limit ourselves exclusively to the first term in expansion (19)

$$\Delta \approx 0.42 f (aD^2)^{-\frac{1}{3}} \delta_0 \quad (20)$$

or

$$\frac{\delta}{\delta_0} = 0.42 \left(\frac{D}{a}\right)^{\frac{1}{3}} = 0.42 \left(\frac{\text{Pr}}{\text{Sc}}\right)^{\frac{1}{3}}. \quad (21)$$

The quantity δ is a strong function of the diffusion capacity of the dissolved substance, of the viscosity and nature of the liquid flow [8] and, generally speaking, of the rate of growth for the solid phase [9]. For the adopted limitation of rather small values for the growth rate, as we can see from Eq. (20), δ is independent of the growth rate. An analogous result for similar limitations was obtained for the case in which the single crystal was extracted from the melt [1]. The relationship between δ and the growth rate f is given by successive approximations of solution (19).

NOTATION

k_0 and k are the equilibrium and effective coefficients of admixture distribution, respectively; δ and δ_0 are the thicknesses of diffusive and hydrodynamic

boundary layers; D is the diffusivity; C_s , C_0 , C_1 , and C_1^0 are the admixture concentrations in a solidified phase at the liquid-solid interface within a diffusive layer and in the bulk of the melt mass, respectively; v is the velocity of the melt flow in a hydrodynamic boundary layer; x is the coordinate in the flow direction; y is the coordinate normal to the liquid flow direction; ν is the kinematic viscosity; a is the thermal diffusivity; g is the gravity acceleration; β is the thermal expansion coefficient; Θ is the temperature head within a hydrodynamic boundary layer; Pr and Sc are the Prandtl and Schmidt numbers, respectively; f is the growth rate for the solid phase.

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